AP Calculus
Stuff You Must Know

Trig Stuff
Identities

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\cos^2 x = \dfrac{1+\cos 2x}{2}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sec x = \dfrac{1}{\cos x}$

$\csc x = \dfrac{1}{\sin x}$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

$\cot(-x) = -\cot x$

$\sec(-x) = \sec x$

$\csc(-x) = -\csc x$

Trig Values

θ	sin θ	cos θ	tan θ
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	∞

Algebra Stuff

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard form: $Ax + By = C$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Differential Calculus
Derivative Formulas and Rules

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$
$$\frac{d}{dx}\left(\sin x\right) = \cos x$$
$$\frac{d}{dx}\left(\cos x\right) = -\sin x$$
$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x$$
$$\frac{d}{dx}\left(\cot x\right) = -\csc^2 x$$

$$\frac{d}{dx}\left(\sec x\right) = \sec x \tan x$$
$$\frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$$
$$\frac{d}{dx}\left(e^x\right) = e^x$$
$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$
$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\log_b x\right) = \frac{1}{x \ln b}$$
$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\left(uv\right) = uv' + vu'$$
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$
$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))g'(x)$$

Uses of the first and second derivative

Curve Sketching

- $y = f(x)$ must be a continuous function on the given interval.
- To find a critical value, set $f'(x) = 0$ or undefined.
- Use a labeled first-derivative chart to determine if the function has a relative max or min. Make sure you write sentences explaining why.
- Alternately, relative max. If $f''(x_0) = -$ you can use the Second Derivative Test. If $f''(x_0) = +$, then x_0 is the x-coordinate of the relative minimum. If $f''(x_0) = -$, then x_0 is the x-coordinate of the relative maximum.
- To find points of inflection, set $f''(x) = 0$ or undefined. Use a labeled second-derivative sign chart to show that the sign of $f''(x)$ changes at that point.

Three Important Theorems

Intermediate Value Theorem

If a function, $f(x)$ is continuous on a closed interval [a, b], and *y* is some value between *f(a)* and *f(b)*, then there exists at least one number *x* = *c* in the open interval (a, b) where $f(c) = y$.

In simple words, the function must pass through every y-value between *f(a)* and *f(b).*

Mean Value Theorem for Derivatives

If the function, $f(x)$ is continuous on the closed interval [a, b] AND $f(x)$ is differentiable on the open interval (a, b), then there exists at least one number *x* = *c* in the open interval (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

In simple words, there is at least one point in the interval (a, b) where the slope of the tangent line is parallel to the secant line drawn through the endpoints of the interval.

Rolle's Theorem

If the function, $f(x)$ is continuous on the closed interval [a, b] AND $f(x)$ is differentiable on the open interval (a, b), AND *f(a)* = *f(b)*, then there exists at least one number *x* = *c* in the open interval (a, b) such that $f'(c) = 0$.

In simple words, if the endpoints of the interval of a differentiable function have the same y-coordinates, then there is at least one point in the interval (a, b) where the slope of the tangent line is equal to zero.
This is really a special case of the Mean Value Theorem.

Integral Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c;\ n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int \sin x\, dx = -\cos x + c$$

$$\int \cos x\, dx \sin x + c$$

$$\int \tan x\, dx = -\ln|\cos x| + c$$

$$\int \cot x\, dx = \ln|\sin x| + c$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + c$$

$$\int \csc x\, dx = -\ln|\csc x + \cot x| + c$$

$$\int \sec^2 x\, dx = \tan x + c$$

$$\int \csc^2 x\, dx = -\cot x + c$$

$$\int \sec x \tan x\, dx = \sec x + c$$

$$\int \csc x \cot x\, dx - \csc x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$$

$$\int \frac{1}{1+x^2} dx = \arctan x + c$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = arc\sec x + c$$

$$\int \ln x\, dx = x\ln x - x + c$$

Fundamental Theorem of Calculus-- Part 1

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

Where $F'(x) = f(x)$

Fundamental Theorem of Calculus-- Part 2

$$\frac{d}{dx}\left(\int_a^x f(t)\, dt\right) = f(x)$$

Average Value Theorem

If the function $f(x)$ is continuous on the closed interval [a, b], there exists some number *c* such that

$$Avg\,Value = f(c) = \frac{1}{b-a}\int_a^b f(x)\, dx$$

Trapezoidal Rule

$$\int_a^b f(x)\, dx \approx \frac{1}{2}\left(\frac{b-a}{n}\right) \cdot$$

$$\left(f(x_0)\right) + 2f(x_1) + \ldots + 2(f(x_{n-1}) + f(x_0))$$

Volume of a Solid of Revolution Disk Method

$$V = \pi\int_a^b \left((OR)^2 - (IR)^2\right) dx\ or\ dy$$

Volume of a Solid of Known Cross-Section

$$V = \int_a^b Area(x)\, dx$$

Particle Motion Formulas

velocity $= \frac{d}{dt}(\text{position})$	accleration $= \frac{d}{dt}(\text{velocity})$	displacement $= \int_{t_1}^{t_2} v(t)\,dt$
total distance $= \int_{t_1}^{t_2} \lvert v(t) \rvert\,dt$	average velocity $= \dfrac{\text{final position - initial position}}{\text{total time}}$	

L'Hopital's Rule

If $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$